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Electromagnetohydrodynamic and Reactive Transport Phenomena in Second Grade Fluids over Moving and Stretching Boundaries in Porous Media under Oscillatory and Unsteady Conditions

[1] Diego Fernando Salazar
[1] Universidad Nacional de Colombia, Colombia

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Abstract

The study of non Newtonian fluids has grown into one of the most important branches of modern fluid mechanics due to its strong relevance in polymer processing, biomedical engineering, geophysical flows, and advanced manufacturing systems. Among the many non Newtonian models, second grade fluids occupy a particularly important position because they are able to represent elastic effects, normal stress differences, and memory behavior while still allowing analytical and semi analytical treatment. When second grade fluids interact with magnetic fields, porous substrates, chemical reactions, and moving or oscillating boundaries, the resulting transport phenomena become highly nonlinear, coupled, and extremely sensitive to both physical and mathematical modeling assumptions. The references forming the basis of this research collectively address magnetohydrodynamic flows, Stokes problems, Couette flows, stretching and shrinking surfaces, porous media, and homotopy based analytical techniques. However, these studies remain scattered across different physical settings and mathematical approaches, often addressing isolated cases rather than offering an integrated theoretical perspective.

The present research develops a unified theoretical framework for understanding unsteady and oscillatory flows of second grade electrically conducting fluids in porous media over moving, stretching, shrinking, and oscillating boundaries with heat and mass transfer and chemical reactions. The framework is built strictly on the ideas, methods, and physical assumptions presented in the provided references, especially those of Cortell, Nazar, Tan and Masuoka, Hayat, Asghar, Yao, Liao, Abbasbandy, Nadeem, Erdogan, Vajravelu, Zeng, and Siddiqui. A comprehensive description of how magnetic fields, porosity, elastic effects, thermal boundary conditions, and surface kinematics interact to shape velocity, temperature, and concentration fields is provided in purely descriptive form, without mathematical expressions.

The homotopy analysis method plays a central role in this work because it allows the construction of convergent series solutions for highly nonlinear boundary layer and Stokes type problems. Drawing from Liao and Abbasbandy, the present study elaborates on how this method reveals the multiplicity of solutions, sensitivity to boundary conditions, and stability features of second grade fluid flows. By synthesizing results from oscillatory wall problems, stretching and shrinking sheet flows, and chemically reactive magnetohydrodynamic transport in porous media, the paper shows that these flows exhibit strong memory dependent behavior, enhanced or suppressed transport depending on magnetic and porous effects, and complex transient dynamics under unsteady forcing.

Keywords: Second grade fluid, magnetohydrodynamics, porous media, homotopy analysis method, unsteady flow, chemical reaction, stretching surface.

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1. Introduction

The mechanics of fluids that do not obey Newtonian constitutive laws has become an indispensable area of scientific inquiry, particularly as technological and industrial systems increasingly rely on complex materials such as polymer melts, biological fluids, suspensions, and electrically conducting liquids. Second grade fluids represent a particularly significant class within the broader family of non Newtonian fluids because they incorporate both viscous and elastic effects while maintaining a constitutive structure that is mathematically tractable and physically interpretable. The classical Newtonian model fails to capture normal stress differences and memory effects that dominate many real materials, whereas second grade fluids provide a minimal extension that includes these features. The foundational works on second grade fluids in unsteady and boundary driven flows, such as those by Erdogan, Asghar, Hayat, and Siddiqui, have shown that even simple boundary motions lead to complex transient behavior not observed in Newtonian systems (Erdogan, 1995; Asghar et al., 2002; Hayat et al., 2009).

One of the most fundamental problems in unsteady fluid mechanics is the Stokes problem, which examines the response of a fluid occupying a half space to the sudden or oscillatory motion of a boundary. While Stokes originally formulated this problem for Newtonian fluids, later extensions to non Newtonian fluids, including second grade fluids, have revealed rich dynamical structures such as elastic waves, memory driven transients, and altered boundary layer development. Nazar and co authors provided new exact solutions for the second Stokes problem in second grade fluids, demonstrating how viscoelasticity modifies the propagation of motion from an oscillating boundary into the fluid domain (Nazar et al., 2010). Similarly, Tan and Masuoka examined the first Stokes problem in a porous half space with a heated boundary, revealing that porosity and thermal effects introduce additional layers of complexity in the transient response (Tan and Masuoka, 2005).

The inclusion of magnetic fields further complicates the flow of electrically conducting second grade fluids. Magnetohydrodynamics introduces Lorentz forces that resist or redirect motion, coupling the velocity field to electromagnetic effects. Cortell studied magnetohydrodynamic flow and mass transfer of a second grade fluid in a porous medium over a stretching sheet with chemically reactive species, highlighting how magnetic fields and reactions jointly influence transport (Cortell, 2007). Hayat and collaborators also investigated exact solutions for aligned magnetohydrodynamic flows of second grade fluids, emphasizing how prescribed velocities and magnetic effects interact with fluid elasticity (Hayat et al., 2009).

Stretching and shrinking surfaces form another major class of problems because they model many industrial processes such as extrusion, coating, and drawing of sheets. Yao and co authors analyzed heat transfer for generalized stretching and shrinking wall problems with convective boundary conditions, showing that thermal behavior is strongly tied to wall kinematics and boundary exchange processes (Yao et al., 2011). When such surfaces interact with second grade fluids in porous and magnetohydrodynamic environments, the resulting flow fields become highly nonlinear and sensitive to both physical and boundary parameters.

A major challenge in all these problems lies in solving the governing equations, which are nonlinear, coupled, and often subject to complex boundary conditions. Traditional perturbation techniques are limited by the requirement of small parameters, which are often absent in real physical systems. Numerical methods, while powerful, may obscure analytical insight and can struggle with stability and convergence in strongly nonlinear regimes. This is where the homotopy analysis method, developed and systematically formalized by Liao, offers a transformative approach. By constructing continuous deformations from simple initial guesses to exact solutions, homotopy analysis provides convergent series representations that are not restricted by small parameters (Liao, 2003; Liao, 2004; Liao, 2009). Abbasbandy and collaborators further extended this framework to predict multiple solutions and analyze their stability, making it particularly suitable for nonlinear boundary value problems arising in fluid mechanics (Abbasbandy and Shivanian, 2010; Abbasbandy et al., 2009).

Despite the richness of the literature, a gap remains in the integration of these diverse strands. Studies on Stokes problems, Couette flows, stretching sheets, porous media, magnetohydrodynamics, and chemical reactions often exist in isolation, focusing on specific configurations or boundary conditions. What is missing is a unified theoretical synthesis that explains how these elements collectively shape the behavior of second grade fluids under unsteady and oscillatory conditions. Moreover, while many papers employ homotopy analysis or related methods, there is limited discussion of how these analytical frameworks reveal deeper physical insights into transport phenomena.

The present research addresses this gap by developing a comprehensive, integrated theoretical narrative based strictly on the provided references. It brings together unsteady and oscillatory boundary driven flows, magnetohydrodynamic and porous effects, heat and mass transfer with chemical reactions, and homotopy based analytical techniques into a single coherent framework. By doing so, it seeks to clarify not only how these physical processes interact, but also why second grade fluids exhibit fundamentally different behavior from Newtonian fluids in these environments.

2. Methodology

The methodological foundation of this research lies in a conceptual synthesis of analytical techniques and physical modeling approaches that have been established in the referenced literature. Rather than introducing new equations or numerical schemes, the methodology is constructed by deeply examining how the governing ideas of second grade fluid mechanics, magnetohydrodynamics, porous media theory, and homotopy based solution techniques are applied and combined.

Second grade fluids are characterized by constitutive relations that include both viscous and elastic terms, capturing the ability of the fluid to store and release energy in response to deformation. In the works of Erdogan, Asghar, and Hayat, this structure is used to model unsteady boundary driven flows where the velocity field depends not only on the instantaneous boundary motion but also on its history (Erdogan, 1995; Asghar et al., 2002; Hayat et al., 2009). The methodology in these studies involves formulating boundary value problems in which the wall motion, whether sudden, oscillatory, or prescribed, imposes a time dependent condition that propagates into the fluid through a combination of viscous diffusion and elastic wave like effects.

When porous media are present, as in the studies of Tan and Masuoka and Cortell, the resistance offered by the porous matrix is incorporated into the momentum balance, effectively damping the flow and altering the penetration depth of motion into the fluid (Tan and Masuoka, 2005; Cortell, 2007). The methodology involves conceptualizing the porous medium as a distributed drag that interacts with both viscous and elastic stresses, leading to modified transient and steady state profiles.

Magnetohydrodynamic effects are introduced by considering the fluid as electrically conducting and subject to an external magnetic field. The Lorentz force generated by the interaction of the induced current with the magnetic field acts as a body force opposing motion, thereby reducing velocity and thickening or thinning boundary layers depending on the configuration. Hayat and Cortell both integrate these effects into their models, showing that magnetic fields provide a powerful control mechanism for second grade fluid flows (Hayat et al., 2009; Cortell, 2007).

Heat and mass transfer are incorporated through energy and species balance considerations, with convective boundary conditions and chemical reactions playing crucial roles. Yao and co authors demonstrated how convective heating at a stretching or shrinking wall influences thermal boundary layers, while Cortell showed how chemically reactive species modify concentration fields (Yao et al., 2011; Cortell, 2007). The methodology here involves coupling the momentum transport with thermal and mass transport, recognizing that velocity fields influence temperature and concentration through convection, while thermal and concentration gradients can in turn affect fluid properties.

The homotopy analysis method serves as the primary analytical framework for exploring these coupled, nonlinear problems. Liao's formulation constructs a continuous deformation from an initial guess to the exact solution through an embedding parameter that controls convergence (Liao, 2003; Liao, 2004). Unlike perturbation methods, this approach does not rely on the presence of small or large parameters. Instead, it introduces auxiliary convergence control parameters that can be adjusted to ensure the resulting series solutions are valid over the entire domain.

Abbasbandy and collaborators extended this methodology to identify multiple solution branches and analyze their stability (Abbasbandy and Shivanian, 2010; Abbasbandy et al., 2009). This is particularly important for stretching and shrinking sheet problems, where multiple physically distinct flow states can exist for the same set of boundary conditions. The methodology thus not only yields solutions but also provides insight into the structure of the solution space.

In problems involving fractional derivatives, reaction diffusion, or micropolar fluids, the homotopy analysis method has also been successfully applied, as shown by Hashim, Bataineh, Nadeem, and others (Hashim et al., 2009; Bataineh et al., 2008; Nadeem et al., 2009). These works demonstrate the versatility of the method and its suitability for highly nonlinear and non classical problems.

By synthesizing these methodological elements, the present research constructs a conceptual approach in which second grade fluid flows under unsteady, oscillatory, magnetohydrodynamic, porous, and reactive conditions are analyzed through homotopy based series solutions. The emphasis is on understanding how convergence control, boundary conditions, and physical parameters jointly determine the qualitative and quantitative features of the flow.

3. Results

The integrated theoretical analysis based on the referenced studies reveals several key patterns in the behavior of second grade fluids under the combined influences of unsteady motion, magnetic fields, porous media, and reactive and thermal effects. One of the most striking results is the profound difference between Newtonian and second grade fluid responses to oscillatory and impulsive boundary motions. In the classical Stokes problem for Newtonian fluids, motion propagates into the fluid purely through viscous diffusion, leading to smooth, exponentially decaying velocity profiles. In contrast, Nazar and Tan and Masuoka showed that second grade fluids exhibit memory driven transients, where elastic effects cause oscillations, phase shifts, and sometimes overshoots in the velocity field (Nazar et al., 2010; Tan and Masuoka, 2005).

When porous media are present, these effects are further modified. The porous matrix introduces an additional resistance that damps both viscous and elastic contributions to the flow. As a result, the penetration of oscillatory motion into the fluid is reduced, and the time required to reach a steady or periodic state is altered. Tan and Masuoka observed that heating at the boundary can counteract some of this damping by changing the effective viscosity and elastic response of the fluid, leading to richer transient behavior (Tan and Masuoka, 2005).

Magnetic fields exert a similarly profound influence. Cortell and Hayat demonstrated that magnetohydrodynamic forces oppose fluid motion and reduce velocity magnitudes, effectively acting as a stabilizing mechanism (Cortell, 2007; Hayat et al., 2009). In second grade fluids, however, this stabilization is not purely monotonic. Elastic effects can interact with magnetic damping to produce complex temporal patterns, including delayed responses and altered boundary layer thicknesses.

In stretching and shrinking sheet problems, Yao and Abbasbandy showed that multiple solution branches can exist, particularly in shrinking configurations where the wall draws fluid inward (Yao et al., 2011; Abbasbandy and Shivanian, 2010). In second grade fluids, elasticity influences which of these solutions are physically realizable by affecting stability and sensitivity to perturbations. The homotopy analysis method reveals these multiple branches through its ability to track different solution trajectories as convergence control parameters are varied.

Heat and mass transfer results further highlight the complexity of these systems. Convective boundary conditions mean that the wall does not impose a fixed temperature or concentration but exchanges heat and mass with the fluid depending on local gradients. Yao showed that this leads to nonlinear coupling between velocity and temperature fields, especially in stretching and shrinking flows (Yao et al., 2011). Cortell's work on chemically reactive species adds another layer, demonstrating that reaction rates can either enhance or suppress concentration boundary layers depending on whether the reaction is generative or consumptive (Cortell, 2007).

The homotopy analysis method results, as reported by Liao, Abbasbandy, and others, consistently show that these nonlinear systems can be represented by convergent series whose behavior depends critically on the choice of auxiliary parameters (Liao, 2009; Abbasbandy et al., 2009). By tuning these parameters, one can capture subtle features such as boundary layer separation, oscillatory decay, and the coexistence of multiple steady states.

4. Discussion

The theoretical findings synthesized in this work underscore the fundamental importance of non Newtonian effects in magnetohydrodynamic and porous media flows. Second grade fluids, by virtue of their elastic memory, respond to boundary forcing in ways that cannot be captured by simpler models. This has profound implications for industrial and technological processes. For example, in polymer extrusion or coating operations involving magnetic fields and porous substrates, ignoring elastic effects could lead to inaccurate predictions of flow stability, heat transfer, and product quality.

One of the key insights from the homotopy analysis based studies is that nonlinear boundary value problems often admit multiple solutions, particularly in shrinking sheet and oscillatory flow configurations. Abbasbandy and Shivanian showed that these multiple solutions correspond to different physical states, some of which may be unstable or physically unrealizable (Abbasbandy and Shivanian, 2010). In second grade fluids, elasticity plays a decisive role in selecting among these states, either stabilizing or destabilizing certain flow patterns.

The interaction between magnetic fields and fluid elasticity is another area of deep theoretical interest. While magnetic damping tends to suppress motion, elastic effects can store and release energy, leading to delayed or oscillatory responses. This interplay suggests the possibility of controlling flow behavior through the combined tuning of magnetic field strength and material properties, a concept that could be exploited in electromagnetic processing of materials.

Chemical reactions and convective boundary conditions introduce additional nonlinearities that further complicate the picture. Cortell and Yao showed that these effects can lead to counterintuitive outcomes, such as increased mass transfer under certain reactive conditions or enhanced heat transfer in shrinking flows (Cortell, 2007; Yao et al., 2011). These phenomena highlight the need for integrated modeling approaches that consider all relevant physical processes simultaneously.

Despite the power of the homotopy analysis method, it is not without limitations. The choice of convergence control parameters requires experience and sometimes trial and error, and the resulting series may converge slowly for certain parameter regimes. Nevertheless, compared to perturbation and purely numerical methods, homotopy analysis offers unmatched insight into the structure of nonlinear solutions and their dependence on physical parameters (Liao, 2003; Liao, 2004).

Future research, building on the foundations laid by the referenced studies, could explore more complex rheological models, three dimensional effects, and transient interactions between multiple moving boundaries. The integration of homotopy analysis with modern computational techniques could also yield hybrid methods that combine analytical insight with numerical robustness.

5. Conclusion

This research has developed a comprehensive theoretical synthesis of unsteady, oscillatory, magnetohydrodynamic, and reactive flows of second grade fluids in porous media over moving, stretching, and shrinking boundaries. By drawing strictly on the provided references, it has shown that the interplay of fluid elasticity, magnetic forces, porous resistance, and boundary driven heat and mass transfer leads to rich and complex flow behavior that cannot be captured by classical Newtonian models.

The homotopy analysis method emerges as a central analytical tool, revealing multiple solution branches, stability features, and subtle nonlinear interactions. The results demonstrate that second grade fluids exhibit memory driven transients, modified boundary layer structures, and sensitivity to both physical and boundary parameters.

These findings not only advance the theoretical understanding of non Newtonian magnetohydrodynamic flows but also have important implications for industrial and technological applications where precise control of flow and transport is required. By integrating diverse strands of the literature into a unified framework, this work provides a foundation for future studies aimed at harnessing the unique properties of second grade fluids in complex environments.

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