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Adaptive Multi Step Differential Transform Analysis of Fractional Order Chaotic and Stiff Dynamical Systems

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Abstract

Fractional order dynamical systems have emerged as one of the most powerful mathematical frameworks for understanding complex physical, biological, and engineering phenomena that exhibit memory, hereditary behavior, and long term dependency. Classical integer order models often fail to capture the full richness of such processes, particularly when chaos, stiffness, or multiscale dynamics are involved. The fractional order Rossler system, in particular, represents one of the most widely studied prototypes for fractional chaotic dynamics because it combines nonlinear feedback, sensitive dependence on initial conditions, and nonlocal temporal effects. However, solving such systems remains a fundamental challenge because fractional derivatives introduce nonlocality, while chaos amplifies numerical errors, and stiffness creates extreme sensitivity to step size. This article develops a comprehensive analytic and numerical investigation of fractional order chaotic and stiff systems using the differential transform method and its multi step and adaptive extensions. Building strictly upon the theoretical and computational foundations established by Zhang and Zhou, Freihat and Momani, Odibat and collaborators, Alomari, Yildirim, Gokdogan, and other contributors, the study examines how generalized and multi step differential transform techniques overcome the core limitations of classical solvers such as Runge Kutta and standard MATLAB routines when applied to chaotic fractional dynamics. The methodology section presents a detailed conceptual explanation of how the differential transform method converts differential equations into recursive algebraic structures that preserve nonlinearity and fractional memory while maintaining computational efficiency. The results demonstrate that multi step and adaptive differential transform strategies achieve superior stability, accuracy, and long time convergence when compared to conventional numerical approaches, particularly in stiff and chaotic regimes. The discussion elaborates on the theoretical implications of these findings for chaos theory, fractional calculus, and numerical analysis, and highlights how the interaction between memory effects and adaptive step control leads to qualitatively different dynamical representations. Limitations of the approach are critically analyzed, including convergence radius, truncation sensitivity, and computational cost. Finally, the article outlines future research directions for hybrid fractional chaos solvers and real world applications in physics, engineering, and biological modeling.

Keywords: Fractional chaos, differential transform method, stiff systems, Rossler dynamics, adaptive numerical analysis, nonlinear oscillators.

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1. Introduction

The study of nonlinear dynamical systems has long occupied a central place in mathematics, physics, and engineering because such systems govern the behavior of real world phenomena ranging from fluid turbulence to neural networks, from chemical reactions to economic cycles. Among these systems, chaotic dynamics occupy a particularly important position because they demonstrate how deterministic equations can produce unpredictable and highly sensitive behavior. The Rossler system, originally proposed as a simple three dimensional chaotic oscillator, has become one of the canonical examples used to explore the nature of chaos, attractors, and nonlinear feedback. When extended into the fractional order domain, the Rossler system gains an additional layer of complexity because fractional derivatives introduce memory effects that fundamentally alter the system's temporal structure and stability properties (Zhang and Zhou, 2009).

Fractional calculus, which generalizes differentiation and integration to non integer orders, provides a powerful framework for modeling processes with long term memory, anomalous diffusion, and hereditary behavior. Such features are ubiquitous in real world systems. In viscoelastic materials, the stress depends not only on the present strain but also on its entire history. In biological systems, population growth and disease spread depend on delayed and accumulated effects. In electrical circuits, fractional elements capture frequency dependent behavior more accurately than integer order components. When fractional calculus is combined with nonlinear dynamics, especially chaotic oscillators, the resulting systems exhibit behaviors that are richer and more subtle than their integer order counterparts (Zhang and Zhou, 2009; Alomari, 2011).

However, this theoretical richness comes at a steep computational cost. Fractional order differential equations are inherently nonlocal, meaning that their evaluation at any time depends on the entire past history of the system. At the same time, chaotic systems amplify numerical errors exponentially, so even small discretization mistakes can grow into large deviations. When stiffness is added, meaning that the system contains components evolving at widely different time scales, the numerical challenge becomes even more severe (Lapidus et al., 1974; Miranker, 1981; Shampine et al., 2003). Traditional solvers, including explicit and implicit Runge Kutta methods such as those implemented in MATLAB, often struggle to maintain stability and accuracy under these combined conditions (Keevil, 2021; Shampine et al., 2003).

These difficulties have motivated the development of alternative numerical and semi analytic techniques designed specifically for nonlinear, stiff, and fractional systems. One of the most important of these techniques is the differential transform method, originally introduced as a systematic procedure for converting differential equations into algebraic recursions that can be solved iteratively (Jang et al., 2000). The core idea is to represent the solution as a power series in time, where the coefficients are determined through transformation rules derived from the governing equations. Because this approach works directly with series coefficients, it avoids the need for discretization in the traditional sense and can achieve high accuracy with relatively few terms when the solution is smooth.

The method was later generalized to fractional order systems by Odibat, Momani, and Erturk, who demonstrated that fractional derivatives can be incorporated into the differential transform framework by modifying the transformation rules to account for non integer orders (Odibat et al., 2008). This generalized differential transform method made it possible to apply the approach to fractional chaotic systems, including the fractional Rossler oscillator, with remarkable success (Freihat and Momani, 2012; Alomari, 2011).

Despite these advances, the classical differential transform method still suffers from important limitations. Because it is based on a single series expansion around an initial point, its radius of convergence may be limited, especially for long time integration or strongly nonlinear dynamics. In chaotic systems, this limitation becomes particularly problematic because the solution may quickly move outside the region where the truncated series provides an accurate approximation. To address this issue, researchers developed the multi step differential transform method, which divides the time domain into multiple subintervals and applies the transform sequentially, using the solution at the end of one subinterval as the initial condition for the next (Odibat et al., 2010; Erturk et al., 2012).

Further refinements introduced adaptive and multistage versions of the method, allowing the step size and series order to be adjusted dynamically in response to the local behavior of the solution (Gokdogan et al., 2012; Yildirim et al., 2012). These developments significantly improved the ability of differential transform techniques to handle stiff and chaotic systems over long time horizons.

The present study situates itself within this rich body of research and seeks to provide a comprehensive theoretical and methodological synthesis of adaptive multi step differential transform analysis applied to fractional order chaotic and stiff systems. While previous studies have demonstrated the feasibility of these methods in specific cases, there remains a need for a deeper theoretical understanding of why and how they succeed, especially in comparison to traditional numerical solvers. Moreover, the interaction between fractional memory, chaos, and stiffness has not been fully explored from a unified methodological perspective.

The central problem addressed in this article is therefore how adaptive multi step differential transform methods can be systematically used to achieve stable, accurate, and physically meaningful solutions for fractional order chaotic systems, particularly the fractional Rossler oscillator, in regimes where classical numerical methods fail or become inefficient. By drawing on the insights of singular perturbation theory (Johnson, 2005; OMalley, 1991; Smith, 2009), stiff system analysis (Lapidus et al., 1974; Miranker, 1981; Miller et al., 1996), and nonlinear dynamics (Khalil, 2002), the study aims to bridge the gap between abstract mathematical theory and practical computational tools.

The literature reveals a clear gap. While many authors have focused either on fractional chaos or on stiff numerical methods, relatively few have examined their intersection in a systematic way. Zhang and Zhou (2009) provided one of the earliest and most influential analyses of chaos in a fractional Rossler system, demonstrating how fractional order alters bifurcation structure and chaotic attractors. Freihat and Momani (2012) applied the differential transform method to fractional chaotic and hyperchaotic Rossler systems, showing that the method can produce accurate numerical analytic solutions. Alomari (2011) further developed analytic solutions for fractional chaotic systems using differential transform techniques. Odibat and colleagues extended the method to multi step and generalized forms, enabling long time integration of chaotic and non chaotic systems (Odibat et al., 2008; Odibat et al., 2010).

At the same time, the broader numerical analysis literature has emphasized the importance of adaptive step size, stiffness handling, and singular perturbation techniques for solving systems with widely varying time scales (Habib and El Zahar, 2008; Gu et al., 2011; Kumar and Parul, 2011). These insights have not always been fully integrated into the fractional chaos community, leading to fragmented approaches.

By synthesizing these strands, this article seeks to provide a unified and theoretically grounded framework for the adaptive multi step differential transform analysis of fractional order chaotic and stiff dynamical systems.

2. Methodology

The methodological foundation of this study rests on the differential transform method and its generalized, multi step, and adaptive variants. To understand how these methods operate in the context of fractional order chaotic systems, it is necessary to examine their conceptual structure in detail.

The classical differential transform method is based on the idea that a sufficiently smooth function can be represented as a power series around a given point in time. Instead of computing derivatives explicitly, the method defines a transformation that maps the original differential equation into a set of algebraic recurrence relations for the series coefficients. Each coefficient corresponds to a derivative of the unknown function evaluated at the expansion point, scaled by a factorial factor. By substituting the series representation into the original equation and equating like powers of time, one obtains a recursive scheme that determines all higher order coefficients from the initial conditions and the governing equations (Jang et al., 2000).

When applied to nonlinear systems, this transformation preserves the nonlinear structure because products and nonlinear functions are handled through convolution type operations on the coefficients. This property makes the method particularly suitable for chaotic systems, where nonlinearity is essential to the dynamics.

The extension to fractional order systems requires modifying the transformation rules to account for fractional derivatives. In fractional calculus, derivatives are defined through integral operators that involve the entire past history of the function. Odibat, Momani, and Erturk (2008) showed that these fractional derivatives can be incorporated into the differential transform framework by introducing generalized transformation rules that relate the fractional derivative of a function to the transformed coefficients through gamma function factors and fractional power indices. This generalized differential transform method allows one to treat fractional order differential equations in a way that is formally similar to the integer order case, while preserving the nonlocal nature of the derivative.

In the context of the fractional Rossler system, the governing equations involve three coupled fractional order differential equations with nonlinear terms. Zhang and Zhou (2009) demonstrated that varying the fractional order parameter significantly alters the system's chaotic behavior, including the onset of chaos and the structure of attractors. Applying the generalized differential transform method to this system involves transforming each equation into a recurrence relation for the coefficients of the three state variables. Freihat and Momani (2012) provided a detailed demonstration of this process, showing that the method yields accurate approximations of the fractional Rossler dynamics over short time intervals.

However, because the solution is expressed as a truncated series, its accuracy is limited to a certain radius of convergence around the expansion point. In chaotic systems, trajectories can move rapidly through state space, and the local series may become inaccurate if the solution moves too far from the expansion point. This is where the multi step differential transform method becomes essential.

The multi step differential transform method divides the time interval of interest into a sequence of smaller subintervals. On each subinterval, the differential transform method is applied with initial conditions taken from the endpoint of the previous subinterval. In effect, the solution is propagated forward in time by a sequence of local series expansions, each valid over a limited range (Odibat et al., 2010; Erturk et al., 2012). This approach dramatically extends the effective time horizon of the method and allows it to track chaotic trajectories over long periods.

The adaptive and multistage extensions further enhance this framework by allowing the size of each subinterval and the order of the series expansion to vary in response to the local behavior of the solution. Gokdogan, Merdan, and Yildirim (2012) showed that adaptive multi step differential transform methods can dynamically adjust the step size based on error estimates, enabling the solver to use small steps in regions of rapid variation and larger steps where the solution is smoother. Yildirim, Gokdogan, and Merdan (2012) applied these ideas specifically to chaotic systems, demonstrating improved accuracy and efficiency.

From a theoretical perspective, these adaptive strategies are closely related to ideas from singular perturbation theory and stiff system analysis. Stiff systems are characterized by the presence of widely separated time scales, which can cause standard explicit methods to become unstable unless extremely small step sizes are used (Lapidus et al., 1974; Miranker, 1981; Miller et al., 1996). Adaptive step size control is therefore essential for efficient and stable integration. By combining the inherent high order accuracy of the differential transform method with adaptive multi step control, the approach effectively addresses the challenges posed by stiffness and chaos simultaneously.

In this study, the methodology is conceptual rather than computational. That is, the analysis focuses on how these methods operate in principle and how they relate to the properties of fractional chaotic systems, rather than presenting explicit numerical experiments or algorithms. This choice is consistent with the aim of providing a theoretical synthesis based strictly on the existing literature.

3. Results

The results of applying adaptive multi step differential transform methods to fractional order chaotic and stiff systems can be understood in terms of several interrelated performance dimensions, including accuracy, stability, convergence, and physical fidelity.

One of the most striking findings reported in the literature is the high level of accuracy achieved by differential transform based methods when applied to fractional chaotic systems. Freihat and Momani (2012) demonstrated that the method produces numerical analytic solutions that closely match reference solutions for fractional Rossler systems, even in hyperchaotic regimes. Alomari (2011) similarly showed that the differential transform method yields highly accurate approximations for fractional chaotic dynamical systems, outperforming many traditional numerical solvers in terms of both precision and computational efficiency.

This accuracy arises from the fact that the method directly approximates the solution as a power series whose coefficients are determined by the governing equations. Unlike finite difference or finite element methods, which approximate derivatives through discrete differences, the differential transform method retains a continuous representation of the solution, at least within each subinterval. This property is particularly valuable in chaotic systems, where small numerical errors can quickly amplify.

Stability is another critical dimension. In stiff systems, numerical instability can arise when the step size is not properly matched to the fastest time scale in the system. Traditional explicit solvers such as classical Runge Kutta methods may require prohibitively small step sizes to remain stable, making them inefficient or even unusable (Shampine et al., 2003; Keevil, 2021). The multi step differential transform method, by contrast, effectively resets the local expansion at each subinterval, preventing the accumulation of instability over long time horizons (Odibat et al., 2010).

The adaptive variants further enhance stability by adjusting the subinterval length in response to the local dynamics. When the system exhibits rapid changes, the method automatically reduces the step size, ensuring that the local series remains within its radius of convergence. When the dynamics are smoother, larger steps can be taken, improving efficiency without sacrificing accuracy (Gokdogan et al., 2012).

Convergence properties are also significantly improved by the multi step and adaptive framework. While a single series expansion may diverge or become inaccurate beyond a certain time, the multi step approach ensures that each expansion is only used within its valid domain. Odibat et al.

(2010) showed that this strategy allows the method to converge for both non chaotic and chaotic systems over long time intervals. In the context of fractional Rossler dynamics, this means that the method can capture the full evolution of chaotic attractors and bifurcations without losing accuracy.

Physical fidelity, meaning the extent to which the numerical solution reflects the true qualitative behavior of the system, is especially important in chaos research. Zhang and Zhou (2009) emphasized that fractional order alters the structure of chaotic attractors and the onset of chaos. A reliable numerical method must therefore reproduce these qualitative features, not just approximate numerical values. The differential transform based methods have been shown to preserve the geometry of attractors and the timing of bifurcations in fractional Rossler systems, indicating that they capture the underlying physics of the system rather than introducing artificial numerical artifacts (Freihat and Momani, 2012; Alomari, 2011).

In comparison with classical solvers, such as those implemented in MATLAB, the literature suggests that differential transform methods often provide more accurate and stable results for chaotic and stiff systems. Keevil (2021) reported that certain fixed step Runge Kutta methods can outperform adaptive solvers like ode45 in some contexts, but these methods still rely on stepwise discretization and may struggle with fractional and stiff dynamics. The differential transform approach, by contrast, operates in a fundamentally different way that is better suited to the nonlocal and multiscale nature of fractional chaos.

4. Discussion

The results summarized above have profound implications for both the theory and practice of modeling fractional order chaotic and stiff systems. At a theoretical level, they demonstrate that the differential transform method and its adaptive multi step extensions provide a bridge between analytic and numerical approaches, offering a way to retain the structural advantages of series solutions while achieving the flexibility and robustness of modern numerical solvers.

One of the most important theoretical insights concerns the relationship between fractional memory and numerical stability. In fractional systems, the present state depends on the entire past, which means that numerical errors can accumulate in a highly nonlocal way. Traditional methods that rely on stepwise updates may inadvertently introduce inconsistencies in this memory, leading to inaccurate or unstable solutions. The differential transform method, by encoding the solution as a series that inherently incorporates the history of the system through its coefficients, offers a more natural representation of fractional dynamics (Odibat et al., 2008; Alomari, 2011).

The multi step and adaptive extensions further enhance this representation by allowing the solver to adapt to changes in the system's behavior. This adaptability is particularly important in chaotic systems, where regions of phase space with smooth dynamics may alternate with regions of rapid variation and sensitivity. By adjusting the step size and series order, the method effectively tracks these changes, maintaining accuracy and stability.

From the perspective of stiff system theory, the success of adaptive multi step differential transform methods can be understood in terms of singular perturbation and multiscale analysis. Stiff systems often contain fast and slow modes, and effective numerical methods must be able to resolve both without excessive computational cost (Johnson, 2005; OMalley, 1991; Smith, 2009). The multi step framework naturally aligns with this requirement by allowing small steps when fast modes dominate and larger steps when the system evolves slowly.

Despite these advantages, it is important to recognize the limitations of the approach. One limitation is the computational cost associated with computing and storing series coefficients, especially for high order expansions and long time simulations. While the method can be efficient in terms of step size, the algebraic complexity of the recurrence relations may become significant for highly nonlinear or high dimensional systems (Odibat et al., 2010; Keimanesh et al., 2011).

Another limitation concerns the choice of truncation order and step size. Although adaptive strategies can mitigate this issue, they rely on error estimates that may themselves be uncertain in chaotic systems. If the truncation order is too low or the step size too large, the local series may fail to capture important dynamics, leading to inaccuracies.

There is also the question of generalizability. While the method has been successfully applied to fractional Rossler systems and other nonlinear oscillators (Erturk et al., 2012; Yildirim et al., 2012), its performance in very high dimensional systems or in systems with discontinuities or noise remains an open area of research.

Future work could explore hybrid approaches that combine differential transform methods with other numerical techniques, such as predictor corrector schemes or homotopy methods (Gu et al., 2011). Such hybrids could further enhance robustness and efficiency, particularly in large scale or real world applications.

5. Conclusion

The adaptive multi step differential transform method represents a powerful and versatile tool for analyzing fractional order chaotic and stiff dynamical systems. By building on the foundational work of Zhang and Zhou, Freihat and Momani, Odibat and colleagues, Alomari, and others, this study has shown how generalized and adaptive differential transform techniques overcome many of the limitations of traditional numerical solvers when applied to fractional chaos.

Through a detailed theoretical analysis, the article has demonstrated that these methods achieve high accuracy, stability, and physical fidelity by combining series based representations with adaptive time stepping. This combination allows the solver to handle the nonlocal memory of fractional derivatives, the sensitivity of chaotic dynamics, and the multiscale nature of stiff systems in a unified and coherent framework.

While challenges remain in terms of computational cost and generalizability, the overall evidence strongly supports the view that adaptive multi step differential transform methods are among the most promising approaches for the simulation and analysis of complex fractional order dynamical systems. As fractional calculus continues to find new applications in science and engineering, these methods are likely to play an increasingly important role in bridging the gap between mathematical theory and practical computation.

References

1. Alomari, A. K. A new analytic solution for fractional chaotic dynamical system using the differential transform method. Computational Mathematics and Applications 2011, 61, 2528 to 2534.
2. Erturk, V. S., Odibat, Z. M., and Momani, S. The multi step differential transform method and its application to determine the solutions of nonlinear oscillators. Advances in Applied Mathematics and Mechanics 2012, 4, 422 to 438.
3. Freihat, A., and Momani, S. Adaptation of differential transform method for the numeric analytic solution of fractional order Rossler chaotic and hyperchaotic systems. Abstract and Applied Analysis 2012.
4. Gokdogan, A., Merdan, M., and Yildirim, A. Adaptive multi step differential transformation method to solving nonlinear differential equations. Mathematical and Computer Modelling 2012, 55, 761 to 769.
5. Gu, W., Liu, W., and Zhang, K. Variable step size prediction correction homotopy method for tracking high dimension Hopf bifurcations. International Journal of Electrical Power and Energy Systems 2011, 33, 1229 to 1235.
6. Habib, H. M., and El Zahar, E. R. Variable step size initial value algorithm for singular perturbation problems using locally exact integration. Applied Mathematics and Computation 2008, 200, 330 to 340.
7. Jang, M. J., Chen, C. L., and Liy, Y. C. On solving the initial value problems using the differential transformation method. Applied Mathematics and Computation 2000, 115, 145 to 160.
8. Johnson, R. S. Singular perturbation theory mathematical and analytical techniques with applications to engineering. Springer Verlag New York 2005.
9. Keevil, J. ODE4 gives more accurate results than ODE45, ODE23, ODE23s. MATLAB Central File Exchange 2021.
10. Keimanesh, M., Rashidi, M. M., Chamkha, A. J., and Jafari, R. Study of a third grade non Newtonian fluid flow between two parallel plates using the multi step differential transform method. Computational Mathematics and Applications 2011, 62, 2871 to 2891.
11. Khalil, H. K. Nonlinear systems. Prentice Hall 2002.
12. Kumar, M., and Parul. Methods for solving singular perturbation problems arising in science and engineering. Mathematical and Computer Modelling 2011, 54, 556 to 575.
13. Lapidus, L., Aiken, R. C., and Liu, Y. A. The occurrence and numerical solution of physical and chemical systems having widely varying time constants. In Stiff Differential Systems, Plenum Press New York 1974, 187 to 200.
14. Miller, J. J. H., ORiordan, E., and Shishkin, G. I. Fitted numerical methods for singular perturbation problems. World Scientific Singapore 1996.
15. Miranker, W. L. Numerical methods for stiff equations and singular perturbation problems. Reidel Dordrecht 1981.
16. Odibat, Z., Momani, S., and Erturk, V. S. Generalized differential transform method application to differential equations of fractional order. Applied Mathematics and Computation 2008, 197, 467 to 477.
17. Odibat, Z., Bertelle, C., Aziz Alaoui, M. A., and Duchamp, G. A multi step differential transform

method and application to non chaotic and chaotic systems. Computational Mathematics and Applications 2010, 59, 1462 to 1472.

18. OMalley, R. E. Singular perturbation methods for ordinary differential equations. Springer Verlag New York 1991.

19. Patra, A., and Saha Ray, S. Multistep differential transform method for numerical solution of classical neutron point kinetic equation. Computational Mathematics and Modeling 2013, 24, 604 to 615.

20. Roos, H. G., Stynes, M., and Tobiska, L. Numerical methods for singularly perturbed differential equations. Springer Berlin 1996.

21. Shampine, L. F., Gladwell, I., and Thompson, S. Solving ODEs with MATLAB. Cambridge University Press 2003.

22. Smith, D. R. Singular perturbation theory an introduction with applications. Cambridge University Press Cambridge 2009.

23. Yildirim, A., Gokdogan, A., and Merdan, M. Chaotic systems via multi step differential transformation method. Canadian Journal of Physics 2012, 90, 391 to 406.

24. Zhang, W., Zhou, S., Li, H., and Zhu, H. Chaos in a fractional order Rossler system. Chaos Solitons and Fractals 2009, 42, 1684 to 1691.