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Nonlinear Transport Phenomena in Viscoelastic and Second Grade Fluids over Stretching Surfaces with Variable Thermophysical Properties and Electromagnetic Effects

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Abstract

The transport of momentum and heat in non-Newtonian fluids over stretching surfaces has become one of the most central themes in modern theoretical and applied fluid mechanics due to its relevance in polymer extrusion, metallurgical processing, coating technologies, biomedical transport, and thermal management systems. Among the many classes of non-Newtonian models, second grade and viscoelastic fluids represent a particularly important family because they simultaneously capture elasticity, normal stress effects, and memory while remaining mathematically tractable for boundary layer analysis. In realistic industrial and biomedical environments, however, fluid properties such as viscosity and thermal conductivity are rarely constant. Instead, they vary strongly with temperature, shear rate, and even concentration of suspended particles. Furthermore, thermal radiation, heat sources and sinks, magnetic fields, and electromagnetic forcing often coexist, especially in high temperature polymer processing, liquid metal transport, microfluidics, and magneto-biofluid applications.

This study develops a comprehensive, theoretically consistent and physically interpretable framework for the coupled flow and heat transfer of second grade and viscoelastic fluids over stretching sheets and plates when viscosity and thermal conductivity vary with temperature and when electromagnetic and radiative effects are present. Building exclusively on the body of literature provided, the work synthesizes results from stretching sheet theory, viscoelastic boundary layer dynamics, magnetohydrodynamics, variable property transport, and biofluidic microtransport into a unified descriptive methodology. The formulation is grounded in the similarity transformation philosophy introduced for stretching surfaces and extended by later authors to non-Newtonian and magnetized flows. The roles of heat generation and absorption, radiation, viscous dissipation, porous substrates, and electromagnetic body forces are all incorporated conceptually.

Rather than presenting equations, this article provides a detailed, narrative-based explanation of how the governing physics, boundary layer structure, and thermodynamic coupling evolve under these complex conditions. Results reported in the literature are reinterpreted to reveal deep physical mechanisms. It is shown that variable viscosity introduces a strong asymmetry between momentum and thermal diffusion layers, while variable thermal conductivity fundamentally alters how heat propagates away from the surface, especially in radiative environments. Second grade elasticity is found to either stabilize or destabilize the flow depending on whether elastic memory reinforces or resists surface stretching. Magnetic fields suppress velocity while enhancing thermal energy retention, a trend that becomes even more pronounced in viscoelastic fluids due to the additional elastic stresses.

Keywords: Viscoelastic fluids, second grade fluids, stretching sheet, variable thermal conductivity, magnetohydrodynamics, heat transfer, non-Newtonian boundary layers.

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1. Introduction

The study of boundary layer flow and heat transfer over stretching surfaces occupies a central place in modern fluid mechanics and thermal engineering because it provides the theoretical basis for understanding how fluids behave when a solid surface is pulled, stretched, or extruded through a surrounding medium. This configuration arises naturally in polymer processing, wire and fiber drawing, continuous casting of metals, glass blowing, extrusion of plastic sheets, and even in biological systems where tissues or membranes undergo deformation. In such systems, the stretching motion of the surface continuously entrains the adjacent fluid, creating a thin boundary layer in which velocity and temperature vary rapidly. The classical theory of stretching sheet flows was originally developed for Newtonian fluids with constant properties, but this idealization fails to describe the real behavior of industrial and biological fluids, which are often non-Newtonian and thermally sensitive.

Among the many non-Newtonian models proposed in the literature, the second grade and viscoelastic fluid models have proven particularly useful. Second grade fluids are characterized by normal stress differences and elastic memory effects, making them capable of describing polymeric liquids, biological fluids, and many suspensions more accurately than Newtonian models. The pioneering stretching sheet analysis for second grade fluids demonstrated that elasticity alters both the momentum and thermal boundary layers in subtle but profound ways, modifying shear stress, heat transfer rates, and flow stability (Vajravelu and Roper, 1999; Cortell, 2006). Subsequent studies showed that these effects become even more complex when thermal radiation, heat sources or sinks, or electromagnetic forces are included (Bataller, 2007; Hayat et al., 2009).

At the same time, researchers recognized that assuming constant viscosity and thermal conductivity can lead to serious inaccuracies when temperature differences are large or when fluids contain additives such as nanoparticles or polymers. Variable thermal conductivity was introduced to account for the fact that heat transport in fluids often becomes more efficient at higher temperatures, particularly in polymer melts and nanofluids (Chiam, 1998; Ahmad et al., 2010). Variable viscosity was similarly incorporated to reflect the strong temperature dependence of resistance to flow, a factor that can dramatically influence boundary layer thickness and shear stress (Soundalgekar et al., 2004; Ramya et al., 2014). When these variable property effects are combined with viscoelasticity, the resulting flow is governed by a delicate balance between elastic stresses, viscous dissipation, and thermal diffusion.

Magnetohydrodynamics adds another layer of complexity and practical relevance. In electrically conducting fluids, magnetic fields induce Lorentz forces that oppose motion and convert kinetic energy into thermal energy. This phenomenon is exploited in electromagnetic pumping, metallurgical processes, and microfluidic control systems (Hatami et al., 2014; Ramos, 2017). When non-Newtonian fluids are subjected to magnetic fields, their elastic and viscous properties interact with electromagnetic forces in ways that can either stabilize or destabilize the flow, depending on the configuration (Abel and Mahesha, 2008; Khan et al., 2012). These effects are particularly important in emerging biomedical technologies, such as magnetically guided drug delivery and mucus clearance in airways, where biofluids are manipulated using external fields (Ally et al., 2008; Siddiqui et al., 2019).

Despite decades of research, the literature remains fragmented. Stretching sheet studies of viscoelastic fluids have traditionally focused on industrial polymer processing, while magneto-biofluid and microfluidic investigations have evolved along a separate trajectory. Variable thermal conductivity and viscosity have been studied extensively in boundary layer contexts, but often without considering electromagnetic effects or elastic memory. The present work addresses this gap by developing a unified theoretical narrative that integrates all these aspects. By synthesizing the results of Chiam (1998), Soundalgekar et al. (2004), Ahmad et al. (2010), Abel and Mahesha (2008), Akinbobola and Okoya (2015), and many others, this article provides a holistic understanding of how non-Newtonian fluids with variable properties behave over stretching surfaces in the presence of radiation, heat sources, porous media, and magnetic fields.

2. Methodology

The methodological framework adopted in this study is entirely grounded in the similarity transformation and boundary layer philosophy that underpins all stretching sheet analyses in the provided references. Instead of presenting mathematical equations, the approach is described conceptually to ensure that the physical meaning of each step is fully transparent.

The starting point is the physical configuration of a flat or slightly curved surface that is being stretched in its own plane with a velocity proportional to distance from a fixed origin. This stretching motion continuously draws fluid from the surrounding medium, creating a steady boundary layer. For second grade and viscoelastic fluids, the stress within this layer is not determined solely by the instantaneous rate of deformation but also by its history. This memory effect is captured in the second grade constitutive relation, which introduces additional normal stresses and modifies the effective resistance to flow (Vajravelu and Roper, 1999; Cortell, 2006).

Heat transfer is coupled to the flow through energy conservation, which accounts for conduction, convection, viscous dissipation, thermal radiation, and internal heat generation or absorption. Variable thermal conductivity means that the ability of the fluid to conduct heat increases or decreases with temperature, a feature that is especially important when large temperature gradients exist near the stretching surface (Chiam, 1998; Ahmad et al., 2010). Variable viscosity similarly allows the momentum diffusion to change with temperature, leading to non-uniform thickening or thinning of the boundary layer (Soundalgekar et al., 2004; Ramya et al., 2014).

When a magnetic field is applied normal to the stretching surface, electrically conducting fluids experience a Lorentz force that acts opposite to the direction of motion. This magnetic damping reduces velocity but generates additional heat through Joule dissipation, thereby coupling the momentum and energy equations even more strongly (Abel and Mahesha, 2008; Khan et al., 2012). In porous media, the presence of a solid matrix introduces an additional drag force, which further modifies the flow structure (Makanda et al., 2013; Chauhan and Olkha, 2012).

The similarity approach reduces this complex system to a set of ordinary differential descriptions by exploiting the self-similar nature of the stretching sheet geometry. Physically, this means that the shape of the velocity and temperature profiles remains the same at all streamwise locations when expressed in appropriately scaled coordinates (Mamaloukas et al., 2007; Akinbobola, 2015). This allows researchers to explore how changes in physical parameters, such as elasticity, magnetic field strength, or thermal conductivity variation, influence the overall behavior of the system.

Numerical and semi-analytical techniques, such as the homotopy perturbation method and Adomian decomposition, have been widely used in the literature to obtain solutions to these similarity equations (Vivek and Aisha, 2014; Manzoor et al., 2019). These methods provide detailed information about velocity, temperature, and stress distributions, which can then be interpreted in terms of physical mechanisms.

3. Results

Across the diverse body of literature considered, several robust trends emerge regarding the influence of viscoelasticity, variable properties, and electromagnetic effects on stretching sheet flows. One of the most consistent findings is that second grade elasticity alters the balance between viscous and inertial forces in the boundary layer. In many cases, elastic stresses resist the deformation imposed by the stretching surface, leading to a reduction in velocity gradients near the wall and a corresponding decrease in shear stress (Vajravelu and Roper, 1999; Cortell, 2006). However, under certain conditions, particularly when the stretching rate is high, elastic memory can enhance momentum transport and thicken the boundary layer, demonstrating the dual nature of viscoelasticity (Bataller, 2007; Bhattacharyya et al., 2011).

Variable viscosity has a pronounced impact on these trends. When viscosity decreases with increasing temperature, the fluid near a hot stretching surface becomes less resistant to flow, resulting in higher velocities and thinner momentum boundary layers (Soundalgekar et al., 2004; Akinbobola and Okoya, 2015). This effect can partially offset the damping introduced by magnetic fields or porous media, leading to complex, non-monotonic behavior.

Variable thermal conductivity strongly influences the temperature field. When thermal conductivity increases with temperature, heat is transported more efficiently away from the surface, leading to a thicker thermal boundary layer and lower surface temperatures (Chiam, 1998; Ahmad et al., 2010). In radiative environments, this enhanced conduction interacts with radiation to produce even more pronounced temperature smoothing (Cortell, 2014; Hayat et al., 2009).

Magnetic fields consistently reduce fluid velocity by introducing Lorentz forces that oppose motion. This magnetic braking increases the residence time of fluid near the hot surface, thereby enhancing heat transfer and raising fluid temperatures (Abel and Mahesha, 2008; Khan et al., 2012). In viscoelastic fluids, the combination of magnetic damping and elastic resistance leads to particularly strong suppression of motion, which can be advantageous in applications such as electromagnetic pumping and flow control (Hatami et al., 2014; Ramos, 2017).

The inclusion of heat sources or sinks further modifies these patterns. Internal heat generation raises fluid temperature and can destabilize the thermal boundary layer, while heat absorption has a cooling effect that stabilizes it (Bataller, 2007; Bhattacharyya et al., 2011). When combined with variable thermal conductivity, these effects become highly nonlinear.

4. Discussion

The results synthesized in this study highlight the intricate interplay between viscoelasticity, variable properties, and electromagnetic forces in stretching sheet flows. From a theoretical perspective, these interactions reveal that no single parameter can be adjusted in isolation without affecting the entire transport process. For example, increasing magnetic field strength may be intended to slow the flow, but in a fluid with temperature-dependent viscosity, the resulting heating can reduce viscosity and partially negate the magnetic damping. Similarly, enhancing thermal conductivity to improve heat removal may alter viscosity and elasticity, leading to unexpected changes in momentum transport.

These complexities are particularly relevant in biomedical and microfluidic applications. In cilia-driven biofluid transport, for example, magnetic fields can be used to guide particles or enhance mucus clearance, but the viscoelastic nature of biological fluids means that elastic stresses and temperature effects must be carefully considered (Siddiqui et al., 2019; Bhatti et al., 2017). The same is true in magnetically assisted drug delivery, where variable viscosity and thermal conductivity can influence particle trajectories and heat generation (Ally et al., 2008; Elkhair et al., 2017).

Despite the progress made, several limitations remain. Most studies rely on similarity solutions that assume idealized geometries and steady conditions. Real systems often involve unsteady, three-dimensional, and curved surfaces, as well as complex boundary conditions (Baris and Dokuz, 2006; Hoque et al., 2013). Moreover, experimental validation of variable property and viscoelastic effects remains limited, highlighting the need for integrated theoretical and experimental research.

5. Conclusion

This comprehensive analysis demonstrates that the flow and heat transfer of viscoelastic and second grade fluids over stretching surfaces are governed by a delicate balance between elastic stresses, variable thermophysical properties, radiation, and electromagnetic forces. Variable viscosity and thermal conductivity fundamentally alter the structure of the boundary layers, while magnetic fields and heat sources provide powerful means of controlling transport processes. By synthesizing the extensive literature on these topics, this study provides a unified theoretical foundation that can guide future research in polymer processing, thermal management, and biomedical fluid dynamics.

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