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Spectral and Transform Based Numerical Frameworks for Nonlinear, Fractional, and Epidemic Differential Systems

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Abstract

This study presents a comprehensive and integrative examination of modern numerical frameworks for solving ordinary, partial, fractional, and epidemiological differential systems using spectral and differential transform based methodologies. Drawing strictly on the referenced body of work, the article investigates how polynomial based spectral schemes, differential transformation methods, and adaptive time stepping strategies collectively form a powerful computational ecosystem for addressing nonlinear and complex dynamical models. Differential equations are fundamental tools for describing heat conduction, epidemic propagation, biological interactions, and wave phenomena, yet the inherent nonlinearity, fractional order behavior, and multi dimensionality of many real world systems continue to challenge traditional numerical solvers. As demonstrated in computational immunology models, ordinary differential equations provide a flexible representation of interacting biological components, but their accuracy depends heavily on the stability and convergence of the underlying numerical method used for simulation (Hoops et al., 2016). Spectral based schemes, including Gegenbauer, Lucas, Legendre, and Chebyshev polynomial families, have emerged as high precision approximators that can dramatically reduce computational cost while maintaining global accuracy for smooth solutions (Sayed et al., 2024; Youssri et al., 2023). In parallel, differential transformation methods have proven to be efficient semi analytical techniques that convert differential models into algebraic recurrence systems, enabling the study of nonlinear epidemic, chaotic, and damped wave models with strong stability and fast convergence (Ahmad et al., 2017; Odibat et al., 2010).

This article synthesizes these two dominant traditions into a unified narrative that highlights their complementary strengths. By examining how polynomial based spectral algorithms manage spatial and fractional complexities while differential transforms manage temporal and nonlinear expansions, this study demonstrates how advanced numerical solvers can be structured to address the limitations of classical finite difference and Runge Kutta methods. The literature further shows that modified polynomial bases and alleviated spectral shifts enhance numerical conditioning, making these approaches suitable for boundary value problems, time fractional equations, and coupled nonlinear systems (Sayed et al., 2024; Abd Elhameed et al., 2023). At the same time, multistep differential transform schemes overcome the local convergence limitation of classical transforms and provide reliable long time simulations for chaotic and epidemiological models (Odibat et al., 2010).

Keywords: Spectral methods, differential transform, fractional equations, epidemic modeling, numerical stability, nonlinear systems.

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1. Introduction

Differential equations remain one of the most powerful mathematical tools for representing real world phenomena across physics, biology, engineering, and social sciences. Whether modeling the spread of infectious diseases, the transfer of heat in solid materials, or the oscillatory behavior of nonlinear waves, differential equations allow scientists to describe change, interaction, and evolution in a rigorous yet flexible form. However, most real systems produce equations that are either nonlinear, fractional, or high dimensional, making them analytically unsolvable. This has led to the development of numerical and semi analytical approaches that approximate solutions with acceptable accuracy and stability. Among the many approaches proposed, spectral methods and differential transformation techniques have become increasingly prominent due to their capacity to capture global behavior using compact mathematical representations.

The importance of these approaches becomes particularly evident when studying biological and epidemiological systems. Computational immunology relies on ordinary differential equations to describe interactions between immune cells, pathogens, and signaling molecules (Hoops et al., 2016). These models often involve stiff and highly nonlinear dynamics, where small changes in parameters can lead to drastically different outcomes. Traditional low order numerical methods may fail to capture such sensitivity without extremely fine discretization, which increases computational cost and reduces efficiency. Spectral methods and differential transform methods, by contrast, offer high order approximations that can represent smooth solution structures with fewer computational resources.

In the context of heat transfer and wave propagation, partial and fractional differential equations introduce additional challenges. Fractional derivatives reflect memory and hereditary properties of materials, making them indispensable for modeling anomalous diffusion, viscoelasticity, and complex media. However, fractional equations are notoriously difficult to solve numerically because they involve nonlocal operators that link every point in time or space to all others. Spectral algorithms based on orthogonal polynomials have demonstrated remarkable performance in this domain by transforming fractional operators into algebraic structures that can be handled efficiently (Sayed et al., 2024; Abd Elhameed et al., 2023).

The differential transformation method represents a different but complementary philosophy. Instead of discretizing the domain, it expands the solution as a power series whose coefficients are computed recursively from the governing equations. This allows the method to bypass discretization error while preserving the analytical structure of the problem (Biazar and Eslami, 2010). For epidemic models such as SIS and SI frameworks, differential transformation provides direct insight into how infection levels evolve over time and how nonlinear interaction terms shape disease dynamics (Ahmad et al., 2017). Multistep versions of the method extend its applicability to chaotic systems and long time simulations, addressing one of the main limitations of classical power series approaches (Odibat et al., 2010).

Despite the growing literature on spectral and transform methods, a significant gap remains in terms of integrated theoretical understanding. Most studies focus on a single equation type or a specific polynomial family, such as Lucas, Chebyshev, or Gegenbauer, without exploring how these methods relate to each other as part of a broader numerical paradigm. Likewise, differential transform methods are often presented as isolated algorithms for specific equations rather than as components of a unified framework for nonlinear dynamics. The absence of a holistic perspective makes it difficult for researchers and practitioners to select and combine methods optimally for complex, multiscale problems.

This article addresses this gap by synthesizing the referenced literature into a comprehensive framework that connects spectral polynomial methods, differential transform techniques, and adaptive Runge Kutta strategies. By doing so, it aims to demonstrate how these approaches collectively overcome the limitations of traditional numerical solvers and provide a powerful toolkit for modern scientific modeling. The focus is not on deriving new formulas but on explaining, in depth and with theoretical rigor, how these methods work, why they are effective, and how they can be integrated to handle nonlinear, fractional, and epidemiological differential systems.

2. Methodology

The methodological foundation of this research is built on a descriptive and integrative analysis of spectral and differential transform based numerical schemes. Rather than presenting algorithmic formulas, the approach emphasizes conceptual clarity and theoretical relationships among methods. This choice reflects the goal of constructing a unified understanding rather than a narrowly technical procedure.

Spectral methods form the first pillar of the methodology. These methods rely on representing the unknown solution of a differential equation as a weighted sum of carefully chosen basis functions. The most common bases are orthogonal polynomials, such as Legendre, Gegenbauer, Lucas, and various kinds of Chebyshev polynomials. Each of these families has specific mathematical properties that make them suitable for different classes of problems. For example, Gegenbauer polynomials generalize both Legendre and Chebyshev polynomials, allowing them to adapt to a wide range of boundary conditions and weight functions (Sayed et al., 2024). Lucas polynomials, when modified appropriately, offer stable approximations for boundary value problems, particularly those involving second order operators (Youssri et al., 2023).

The key idea of spectral methods is global approximation. Unlike finite difference schemes, which approximate derivatives locally by comparing neighboring grid points, spectral methods use information from the entire domain to construct the solution. This global perspective leads to exponential convergence for smooth solutions, meaning that accuracy increases extremely rapidly as the number of basis functions increases. This property is crucial for problems such as heat conduction and fractional diffusion, where the solution is often smooth but influenced by long range interactions (Atta et al., 2022; Abd Elhameed et al., 2023).

To apply spectral methods to a given differential equation, the equation is first projected onto the chosen polynomial basis. This projection transforms the continuous problem into a system of algebraic equations for the coefficients of the polynomial expansion. When dealing with fractional or nonlinear terms, specialized techniques such as shifted bases and alleviated spectral strategies are used to maintain numerical stability and accuracy (Sayed et al., 2024). These modifications reduce the sensitivity of the system to boundary layers and high order derivatives, which are common in fractional models.

The second pillar of the methodology is the differential transformation method. This approach converts differential equations into recurrence relations for the coefficients of a power series representation of the solution. Each coefficient is determined directly from the structure of the differential equation, making the method semi analytical in nature (Biazar and Eslami, 2010). For nonlinear equations, the transformation rules allow nonlinear terms to be expressed in terms of previously computed coefficients, enabling efficient computation without linearization.

Differential transformation is particularly effective for initial value problems and time dependent systems. In epidemic modeling, for example, the number of susceptible and infected individuals evolves according to nonlinear interaction terms that are naturally captured by power series expansions (Ahmad et al., 2017). However, classical differential transformation suffers from a limited radius of convergence, meaning that it may only provide accurate solutions over a short time interval. To overcome this, multistep differential transformation methods divide the time domain into segments and apply the transformation repeatedly, ensuring global accuracy over long time periods (Odibat et al., 2010).

The methodology also incorporates adaptive Runge Kutta strategies as a reference and validation tool. Fehlberg's low order Runge Kutta formulas with step size control are widely used to benchmark numerical solutions and ensure stability in stiff systems (Fehlberg, 1965). Although Runge Kutta methods are not as globally accurate as spectral or transform methods, their adaptive step size allows them to handle rapid changes in the solution, making them useful for verifying the correctness of more advanced schemes.

By integrating these three methodological components, the study creates a robust framework for analyzing differential systems. Spectral methods handle spatial and fractional complexity, differential transformation methods manage temporal and nonlinear dynamics, and adaptive Runge Kutta methods provide stability control and validation. This integrated approach reflects the way modern computational science increasingly relies on hybrid numerical strategies rather than single isolated algorithms.

3. Results

The results emerging from the synthesis of the referenced studies demonstrate the remarkable effectiveness of spectral and differential transform based methods in solving a wide range of differential equations. One of the most significant findings is the consistently high accuracy achieved by polynomial spectral algorithms when applied to boundary value problems and fractional equations. Modified Lucas polynomials, for instance, have been shown to provide stable and precise solutions for second order boundary value problems, even when the underlying equations exhibit strong nonlinearity (Youssri et al., 2023). This indicates that carefully designed polynomial bases can neutralize numerical instability that would otherwise arise from steep gradients or boundary layers.

Similarly, Gegenbauer and Legendre based spectral methods have been found to be particularly effective for fractional differential equations, where classical discretization techniques often struggle. The alleviated shifted Gegenbauer method improves conditioning by adjusting the polynomial basis to better match the behavior of fractional operators, resulting in smoother coefficient distributions and more reliable convergence (Sayed et al., 2024). In practical terms, this means that fewer basis functions are needed to achieve a given level of accuracy, reducing computational cost without sacrificing precision.

Chebyshev based algorithms, especially those using higher order polynomial families such as the eighth and fifth kinds, have demonstrated strong performance in nonlinear time fractional equations and heat conduction models (Abd Elhameed et al., 2023; Atta et al., 2022). These results confirm that the choice of polynomial family is not merely a technical detail but a critical factor in determining numerical stability and efficiency. By selecting a basis that aligns with the physical and mathematical properties of the problem, researchers can significantly enhance solution quality.

The differential transformation method yields equally compelling results, particularly in the context of epidemic and chaotic systems. For SIS and SI epidemic models, the method produces rapidly convergent series solutions that accurately track infection dynamics over time (Ahmad et al., 2017). This allows for detailed analysis of how transmission rates and recovery processes interact, which is essential for public health modeling. The method also handles quadratic and Riccati type nonlinearities with ease, demonstrating its versatility for a broad class of problems (Biazar and Eslami, 2010).

Multistep differential transformation methods extend these benefits to long time and chaotic systems. By repeatedly reinitializing the power series over successive time intervals, the method maintains accuracy even when the solution exhibits sensitive dependence on initial conditions (Odibat et al., 2010). This is particularly important for nonlinear dynamical systems where small numerical errors can grow exponentially if not controlled.

When compared conceptually to adaptive Runge Kutta methods, spectral and differential transform techniques show superior global accuracy for smooth problems. Fehlberg's adaptive Runge Kutta approach remains valuable for handling stiffness and abrupt changes, but it requires many small steps to achieve the same level of accuracy that spectral methods obtain with a compact representation (Fehlberg, 1965). The combined use of these methods therefore provides both reliability and efficiency.

4. Discussion

The findings of this study highlight a fundamental shift in how numerical differential equations are approached in modern computational science. Rather than relying solely on local discretization techniques, researchers increasingly favor global approximation and semi analytical methods that capture the underlying structure of the problem. Spectral and differential transform methods exemplify this shift by leveraging the mathematical properties of polynomials and power series to achieve high accuracy with relatively low computational effort.

One of the most important theoretical implications is the role of orthogonality and basis selection. The success of Legendre, Gegenbauer, Lucas, and Chebyshev polynomials demonstrates that numerical stability is not an abstract property but a direct consequence of how well the basis functions align with the solution space of the differential equation (Sayed et al., 2024; Youssri et al., 2023). This suggests that future research should focus not only on developing new algorithms but also on identifying optimal bases for specific classes of problems.

The differential transformation method offers a complementary perspective by preserving the analytical form of the equation. This is particularly valuable for nonlinear and epidemiological models, where understanding the role of interaction terms is just as important as computing numerical values (Ahmad et al., 2017). By providing series representations of the solution, the method allows researchers to analyze how different terms contribute to system behavior, which can inform both theory and application.

Despite these strengths, limitations remain. Spectral methods require smooth solutions and may struggle with discontinuities or sharp fronts unless special techniques are employed. Differential transformation methods, even in multistep form, may become computationally expensive for very high dimensional systems. These limitations point to the need for hybrid approaches that combine the strengths of different methods while mitigating their weaknesses.

Future research should therefore explore adaptive and hybrid frameworks that dynamically select between spectral, transform, and Runge Kutta techniques based on the evolving behavior of the solution. Such an approach would reflect the complexity of real world systems, which often exhibit both smooth and rapidly changing dynamics.

5. Conclusion

This article has provided a comprehensive and theoretically grounded examination of spectral and differential transform based numerical methods for solving nonlinear, fractional, and epidemiological differential systems. By synthesizing the referenced literature, it has shown that these methods are not isolated tools but components of a coherent computational paradigm that emphasizes global approximation, analytical structure, and numerical stability. The findings confirm that polynomial spectral schemes and differential transformation techniques offer powerful alternatives to traditional discretization based solvers, particularly for complex and multiscale problems. As scientific modeling continues to grow in complexity, these methods will play an increasingly central role in enabling accurate, efficient, and insightful simulations.

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